1 Class V: lag length determination, extensions, criticisms

1.1 Lag length determination

Can use a LR test:

$$\phi_{LR} = T\left[\ln|\hat{\mathbf{\Sigma}}_{p-1}| - \ln|\hat{\mathbf{\Sigma}}_{p}|\right] \underset{(a)}{\overset{H_0}{\sim}} \chi^2_{n^2} \tag{1}$$

where $\hat{\mathbf{\Sigma}}_l$ is residual covariance matrix estimated from VAR with l lags. Remember we can also used small sample size correction:

$$\phi_{LR} = (T - k_1)\left[\ln|\hat{\mathbf{\Sigma}}_{p-1}| - \ln|\hat{\mathbf{\Sigma}}_p|\right] \underset{(a)}{\overset{H_0}{\sim}} \chi^2_{n^2} \qquad (2)$$

$$k_1 = \#\text{ regressors under } H_1$$

or can use an *information criterion* (see Lutkepohl, 1993)

An lag order determination information criterion works as follows: choose lag order p^* that minimises:

$$c(p) = \ln|\widehat{\mathbf{\Sigma}}_p| + f(T, k(p)) \qquad (3)$$

$$k = \#\text{ free parameters in VAR(p) model}$$

Sort of goodness of fit criterion penalised for number of parameters. Most widely IC are:

criterion	$f(T,k)$
Akaike (AIC)	$\frac{2}{T}n^2p$
Hannan and Quinn (HQ)	$\frac{2}{T}(\ln(\ln(T)))n^2p$
Schwarz (BIC)	$\frac{1}{T}(\ln(T))n^2p$

All of them are deliver asymptotically correct results. Finite sample properties are dicy.

1.2 Extensions

1.2.1 Identification through heteroskedasticity

(Rigobon 2002)

Take bivariate example:

$$\begin{aligned} \mathbf{A}\varepsilon_t &= \mathbf{F}\mathbf{e}_t && (4) \\ \varepsilon_t &\backsim VWN(0, \mathbf{\Sigma}) && (5) \\ \mathbf{e}_t &\backsim VWN(0, \mathbf{I}_2) && (6) \end{aligned}$$

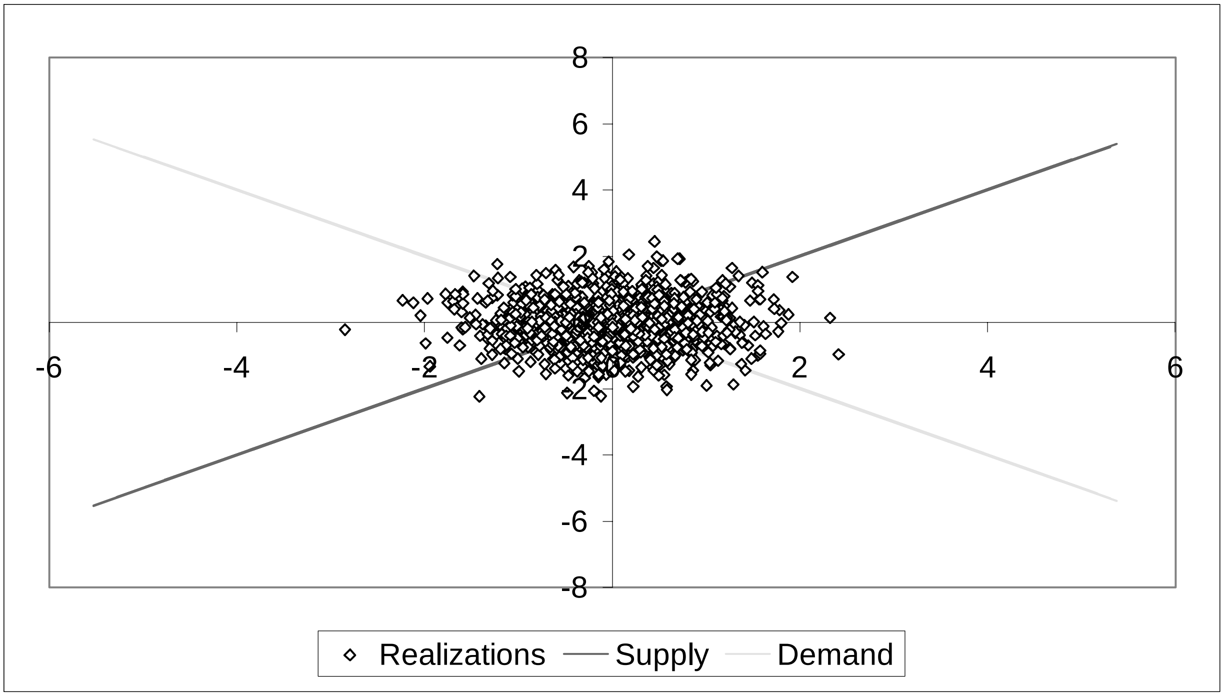
where:

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix} && (7) \\ \mathbf{F} &= \begin{bmatrix} f_{11} & 0 \\ 0 & f_{22} \end{bmatrix} && (8) \end{aligned}$$

This is not identified unless:

(zero) restrictions on a_{12} or a_{21}

sign restrictions?



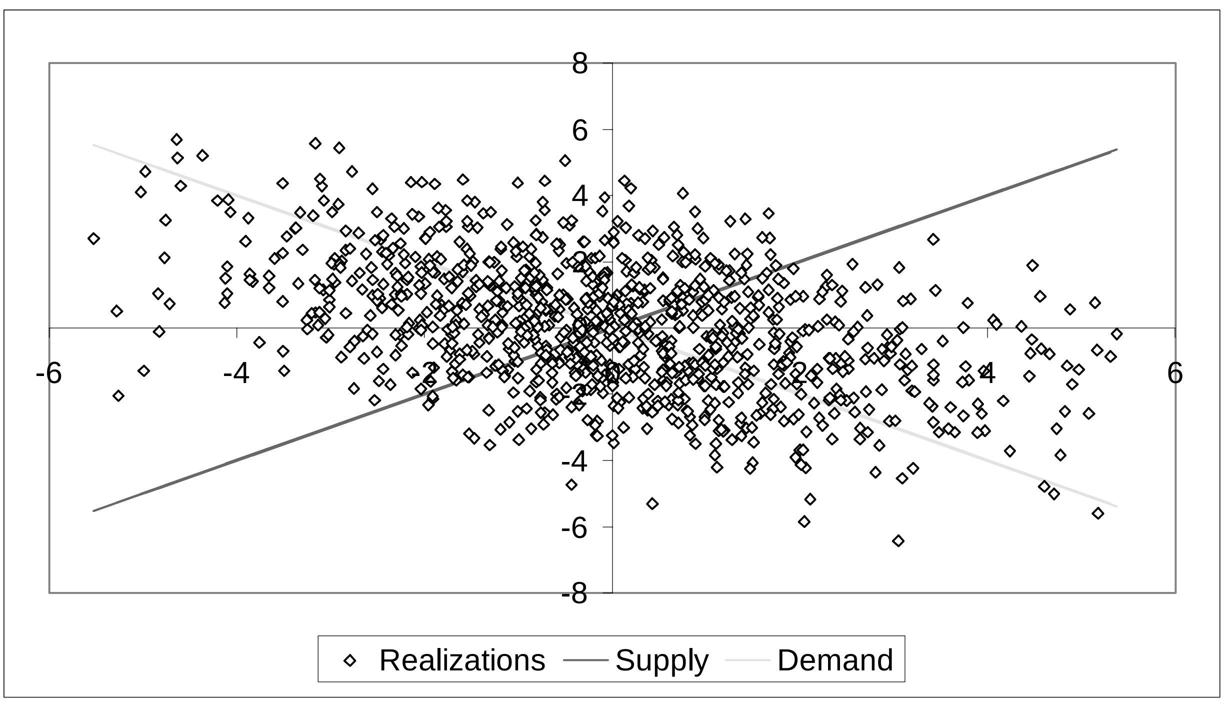


Figure 1: Identification Problem.

constraints on $\frac{f_{11}}{f_{22}}$ (ie when it goes to zero).

In fact (order conditions) suggest that we have 3 equations $(\sigma_{11}, \sigma_{22}, \sigma_{12})$ and 4 unknowns.

But imagine that we have two regimes:

first sub-sample:

$$\begin{aligned}
\mathbf{A}\varepsilon_t &= \mathbf{F}^{(1)}\mathbf{e}_t && (9)\\
\varepsilon_t &\backsim VWN(0, \mathbf{\Sigma}_i) && (10)\\
\mathbf{e}_t &\backsim VWN(0, \mathbf{I}_2) && (11)\\
t &= 1, 2, ...T_1 && (12)
\end{aligned}$$

second sub-sample:

$$
\begin{aligned}
\mathbf{A}\varepsilon_t &= \mathbf{F}^{(2)}\mathbf{e}_t &(13)\\
\varepsilon_t &\backsim VWN(0, \mathbf{\Sigma}^{(2)}) &(14)\\
\mathbf{e}_t &\backsim VWN(0, \mathbf{I}_2) &(15)\\
t &= T_1 + 1, 2, ...T_1 + T_2 &(16)
\end{aligned}
$$

Note that $\mathbf{A}$ is supposed stable across regimes. Note also that timing of regime switch is known.

There are 6 unknowns:

$$(a_{12}, a_{21}, f_{11}^{(1)}, f_{22}^{(1)}, f_{11}^{(2)}, f_{22}^{(2)})$$

and 6 relatioships (for $\sigma_{11}^{(1)}, \sigma_{22}^{(1)}, \sigma_{12}^{(1)}, \sigma_{11}^{(2)}, \sigma_{22}^{(2)}, \sigma_{12}^{(2)}$):

$$\mathbf{A}\boldsymbol{\Sigma}^{(1)}\mathbf{A}' = \mathbf{F}^{(1)}\mathbf{F}^{(1)'} \tag{17}$$

$$\mathbf{A}\boldsymbol{\Sigma}^{(2)}\mathbf{A}' = \mathbf{F}^{(2)}\mathbf{F}^{(2)'} \tag{18}$$

Some algebra (see Rigobon 2002) shows us that a_{12} and a_{21} must satisfy:

$$a_{12} = -\frac{\sigma_{12}^{(i)} + a_{21}\sigma_{11}^{(i)}}{\sigma_{22}^{(i)} + a_{21}\sigma_{12}^{(i)}} \tag{19}$$

$$i = 1, 2$$

and a_{21} satisfies:

$$k_2 a_{21} - k_1 a_{21} + k_0 = 0 \tag{20}$$

$$k_2 = \sigma_{11}^{(1)}\sigma_{12}^{(2)} - \sigma_{12}^{(1)}\sigma_{11}^{(2)} \tag{21}$$

$$k_1 = \sigma_{22}^{(1)}\sigma_{12}^{(2)} - \sigma_{22}^{(2)}\sigma_{11}^{(1)} \tag{22}$$

$$k_0 = \sigma_{12}^{(1)}\sigma_{22}^{(2)} - \sigma_{22}^{(1)}\sigma_{12}^{(2)} \tag{23}$$

Easy to see that if a_{12} and a_{21} solve these equations, also

$$a^*_{12} = \frac{1}{a_{12}} \tag{24}$$

$$a^*_{21} = \frac{1}{a_{21}} \tag{25}$$

solve the same equation (identification up to a row permutation of the original model).

Result: if:

$$\left| \mathbf{\Sigma}^{(2)} - \frac{\sigma^{(2)}_{11}}{\sigma^{(1)}_{11}} \mathbf{\Sigma}^{(1)} \right| \neq 0 \tag{26}$$

i.e.:

$$\frac{\sigma^{(1)}_{11}}{\sigma^{(1)}_{11}} \neq \frac{\sigma^{(1)}_{12}}{\sigma^{(2)}_{12}} \tag{27}$$

We need that to exclude that the the matrices are proportional (see figure 1 in Rigobon 2002 paper).

1.2.2 Factor augmented VAR model (FAVAR)

Bernanke, Boivin and Eliasz (2004, NBER wp # 10220).

How to correctly identify effects of monetary policy shocks?

$\mathbf{y}_t = (m \times 1)$ vector of observable variables particularly important (eg policy rate)

$\mathbf{x}_t = (n \times 1)$large vector of relevant time series with factor structure:

$$\begin{aligned} \mathbf{x}_t &= \underset{(n\times k)}{\mathbf{\Lambda}_f} \underset{(k\times 1)}{\mathbf{f}_t} + \underset{(n\times m)}{\mathbf{\Lambda}_y} \underset{(m\times 1)}{\mathbf{y}_t} + \underset{(n\times 1)}{\boldsymbol{\eta}_t} \qquad (28) \\ m+k &\ll n \end{aligned}$$

Dynamic factor model (Stock and Watson, 1998, NBER wp # 6702).

Define FAVAR model as:

$$
\begin{aligned}
\mathbf{A}(L)\mathbf{z}_t &= \varepsilon_t \\
\mathbf{z}_t &= \begin{bmatrix} \mathbf{f}_t \\ \mathbf{y}_t \end{bmatrix}
\end{aligned}
$$

How to estimate the model: two step approach:

a) first estimate the factors (via principal components)

b) plug estimates of factors into VAR model

Identification of structural shocks? put $\mathbf{y}_t = FFR_t$ last (m=1).

1.3 Interesting examples of SVAR applications

- Favero (2001) surveys monetary policy analysis VARs and different specifications.

- Christiano, Eichenbaum and Evans (1998, NBER wp # 6400)

- Gali (1999, AER): brings in interesting concept of conditional correlation (conditional on e_i being the only source of shocks in the system:

$$
\begin{aligned}
cor(y_{1t}, y_{2t}|i) &= \frac{\sum_{j=0}^{\infty} c_{1j,i} \cdot c_{2j,i}}{\sqrt{V(y_{1t}|j)V(y_{2t}|j)}} \qquad (29) \\
V(y_{1t}|j) &= \sum_{i=0}^{\infty} c_{1j,i}^2, V(y_{2t}|j) = \sum_{i=0}^{\infty} c_{2j,i}^2
\end{aligned}
$$

(remember FEVD coefficients?)

1.4 Criticisms

- Lippi and Reichlin criticism (Lippi and Reichlin, AER 2003, J of Econometrics, 1994)⇒relevance of non-fundamental representations.

- Faust and Leeper (1997, JBES) : long run constraints are not reliable since $\mathbf{B}(1)$ very imprecisely estimated.

- Why do we need as many structural shocks as variables?$\Rightarrow$Factor models. See Forni, Lippi and Reichlin ("Opening the black box: Structural Factor Models versus Structural VARs", on http://www.eabcn.org)

1.5 Estimation problems

VAR are very over-parameterised and hence estimates inefficient. Possible solutions are to impose constraints or use Bayesian approach: see for instance:

- Leeper, Sims and Zha (1996, Brooking Papers on Economic Activity).

- Fabio Canova's work (http://www.eabcn.org/agenda/training/milan_2004/index.htm