GEMAFI 2006: An introduction to applied DSGE modelling

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Contents

1 A brief description of DSGE model

- Dynamic: t-1,t, expectations
- Stochastic: impulse+propagation
- GE: hholds, firms, monetary authorities
- Possible to assess:
- dynamic effects of different shocks
- their relative importance

- run counterfactuals
- simulate policy manouevres
- Possible to introduce realistic mechanisms (frictions etc..)

1.1 Drawbacks

- not possible to obtain analytical solutions
- scaling up of models very difficult
- awkward estimation

2 Plan of the talk(s)

- basic DSGE model (RBC): specification, solution, estimation (today)
- some more interesting mechanisms (next time)
- asset pricing (next time)

2.1 3 entry-literature slide

Campbell (1994)

ch. 5 Walsh, readable and self containd

Woodford (2003)

3 DSGE models estimation

- Latent variables
- High non-linearity (in the "deep" parameters)
- Almost always solution is only approximated
- Troubles in identification

see Ruge-Murcia (2003), Canova and Sala (2006), Amisano and Tristani (2006a,b), Colacito et al. (2004)

4 Inspecting the mechanism: the basic RBC model

Simple RBC model (Campbell, JME, 1994)

$$
\begin{aligned}
Y_t &= (A_t N_t)^{\alpha} K_t^{1-\alpha} && (1)\\
N_t &= 1 && (2)\\
a_t &= \ln(A_t) = \phi a_{t-1} + \sigma e_t && (3)\\
e_t &\backsim NID(0,1) && (4)\\
K_{t+1} &= (1-\delta)K_t + Y_t - C_t && (5)\\
U(C_t) &= \frac{C_t^{1-\gamma} - 1}{1-\gamma} && (6)\\
U &= \sum_{i=0}^{\infty} \beta^i C_{t+i} && (7)
\end{aligned}
$$

complete markets, AD securities

Assume steady state growth with gross steady state rate

$$G = \frac{Y_{t+1}^{ss}}{Y_t^{ss}} = \exp(g)$$

Define gross return of undepreciated capital

$$R_{t+1} = \alpha \left(\frac{A_{t+1}}{K_{t+1}} \right)^{\alpha} + (1 - \delta) \quad (8)$$

FOC

$$C_t^{-\gamma} = \beta E_t \left[C_{t+1}^{-\gamma} \times R_{t+1} \right] \quad (9)$$

in equilibrium

$$G^{\gamma} = \beta R \Rightarrow g = \frac{\ln(\beta) + r}{\gamma} \tag{10}$$

$$\frac{A}{K} \approx \left(\frac{r+\delta}{1-\alpha}\right)^{1-\alpha} \tag{11}$$

values of parameters? calibration, of course!

$$\begin{aligned} \alpha &= 2/3 \\ g &= .005 \\ r &= .0015 \\ & \sigma?, \phi? \beta? \gamma? \end{aligned}$$

Now, let's log-linearise

$$\begin{aligned} y_t &= \alpha a_t + (1-\alpha)k_t & (12) \\ (1-\phi L)a_t &= \sigma e_t & (13) \\ k_{t+1} &\approx \lambda_1 k_t + \lambda_2 a_t + (1-\lambda_1-\lambda_2)c_t & (14) \\ r_{t+1} &\approx \lambda_3(a_{t+1}-k_{t+1}) & (15) \\ E_t \Delta c_{t+1} &= \frac{\lambda_3}{\gamma} E_t(a_{t+1}-k_{t+1}) & (16) \\ \lambda_1 &= \frac{1+r}{1+g}, \lambda_2 = \alpha \frac{r+\delta}{(1-\alpha)(1+g)}, \lambda_3 = \alpha \frac{r+\delta}{1+r} \end{aligned}$$

To find solution: how to get rid of expectations?

Simplest way:method of undertermined coefficients

guess

$$c_t = \eta_{ca} + a_t + \eta_{ck} k_t \tag{17}$$

substitute into eqs for capital and exptected Δc_{t+1}:

$$
\begin{aligned}
k_{t+1} &= \eta_{kk} k_t + \eta_{ka} a_t && (18)\\
\eta_{ck} \Delta k_{t+1} + \eta_{ca} E_t \Delta a_{t+1} &= \frac{\lambda_3}{\gamma} E_t a_{t+1} - \frac{\lambda_3}{\gamma} k_{t+1} && (19)\\
\eta_{kk} &= \lambda_1 + (1 - \lambda_1 - \lambda_2)\eta_{ck}, && (20)\\
\eta_{ka} &= \lambda_2 + (1 - \lambda_1 - \lambda_2)\eta_{ca} && (21)
\end{aligned}
$$

Substitute (18) and (20) into (19)

$$
\begin{aligned}
& \eta_{ck}\left[\lambda_1 - 1 + (1-\lambda_1-\lambda_2)\eta_{ck}\right] k_t \\
& +\eta_{ck}\left[\lambda_2 + (1-\lambda_1-\lambda_2)\eta_{ca} + \eta_{ca}(\phi - 1)\right] a_t \\
= \; & \frac{\lambda_3}{\gamma}\phi a_t - \frac{\lambda_3}{\gamma}\left[\lambda_1 + (1-\lambda_1-\lambda_2)\eta_{ck}\right] k_t + \\
& -\frac{\lambda_3}{\gamma}\left[\lambda_2 + (1-\lambda_1-\lambda_2)\eta_{ca}\right] a_t
\end{aligned}
$$

Equate coeffs on k_t to find η_{ck}, then find η_{ca} by equating coeffs on a_t

η_{ck} equation take stable root

Hence linear system

$$\begin{aligned} y_t &= \alpha a_t + (1-\alpha) k_t && (22) \\ c_t &= \eta_{ck} k_t + \eta_{ca} a_t && (23) \\ k_{t+1} &= \eta_{kk} k_t + \eta_{ka} a_t && (24) \end{aligned}$$

(Campbell, 1994, section 2.3)

where the elasticities $\eta_{ck}, \eta_{ca}, \eta_{kk}, \eta_{ka}$, are functions of the parameters of the model.

4.1 Basic time series implications

$$
\begin{aligned}
k_{t+1} &= \frac{\eta_{ka}\sigma e_t}{(1-\eta_{kk}L)(1-\phi L)} && (25)\\
a_t &= \frac{\sigma e_t}{(1-\phi L)} && (26)\\
y_t &= \frac{\alpha + [(1-\alpha)\eta_{ka} - \alpha\eta_{kk}]L}{(1-\eta_{kk}L)(1-\phi L)}\sigma e_t && (27)\\
c_t &= \frac{\eta_{ca} + (\eta_{ck}\eta_{ka} - \eta_{ca}\eta_{kk})L}{(1-\eta_{kk}L)(1-\phi L)}\sigma e_t && (28)
\end{aligned}
$$

More general solution approach: use perturbation methods (next time)

Impulse....Propagation

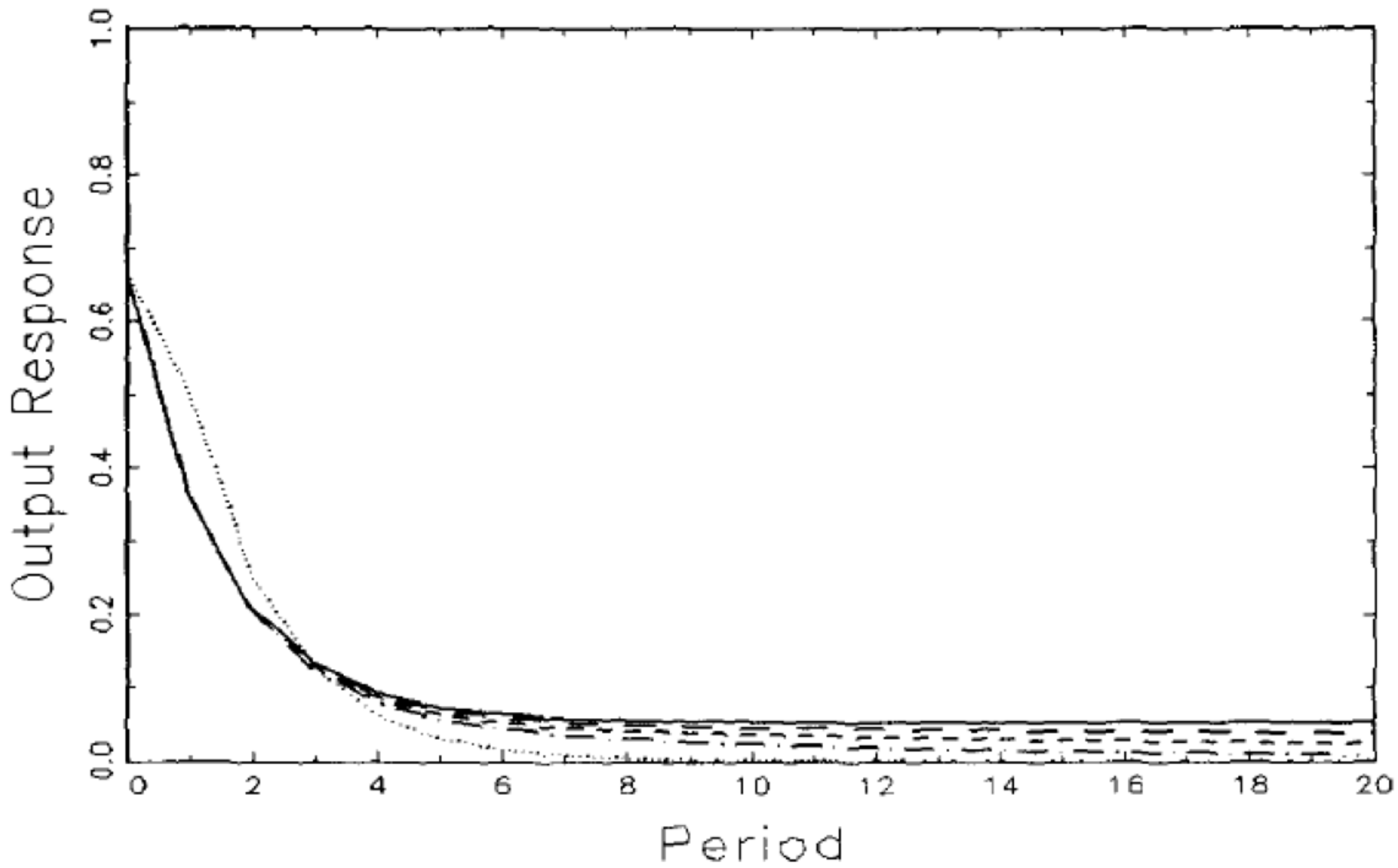

Fig. 1. Output response to a technology shock with fixed labor supply and $\phi = 0.5$.

The solid line gives the percentage response of output to a 1% technology shock in a model with fixed labor supply, specified in eqs. (11), (13), (17), and (18), when the intertemporal elasticity of substitution $\sigma = 0$. The long-dashed line gives the response when $\sigma = 0.2$. The short-dashed line gives the response when $\sigma = 1$. The dashed and dotted line gives the response when $\sigma = 5$. The dotted line gives the response when $\sigma = \infty$. In all cases initial response is $\alpha = 0.667$.

Figure 1

5 Estimation (please be patient...)

Different avenues to estimation, all bumpy

Model falsification

Limited information CEE : take (some) model implied and the empirical (SVAR based) IRFs and minimise weighted distance

Use likelihood based approach

Big issue: identification (Canova and Sala, 2006, Amisano and Tristani 2006a)

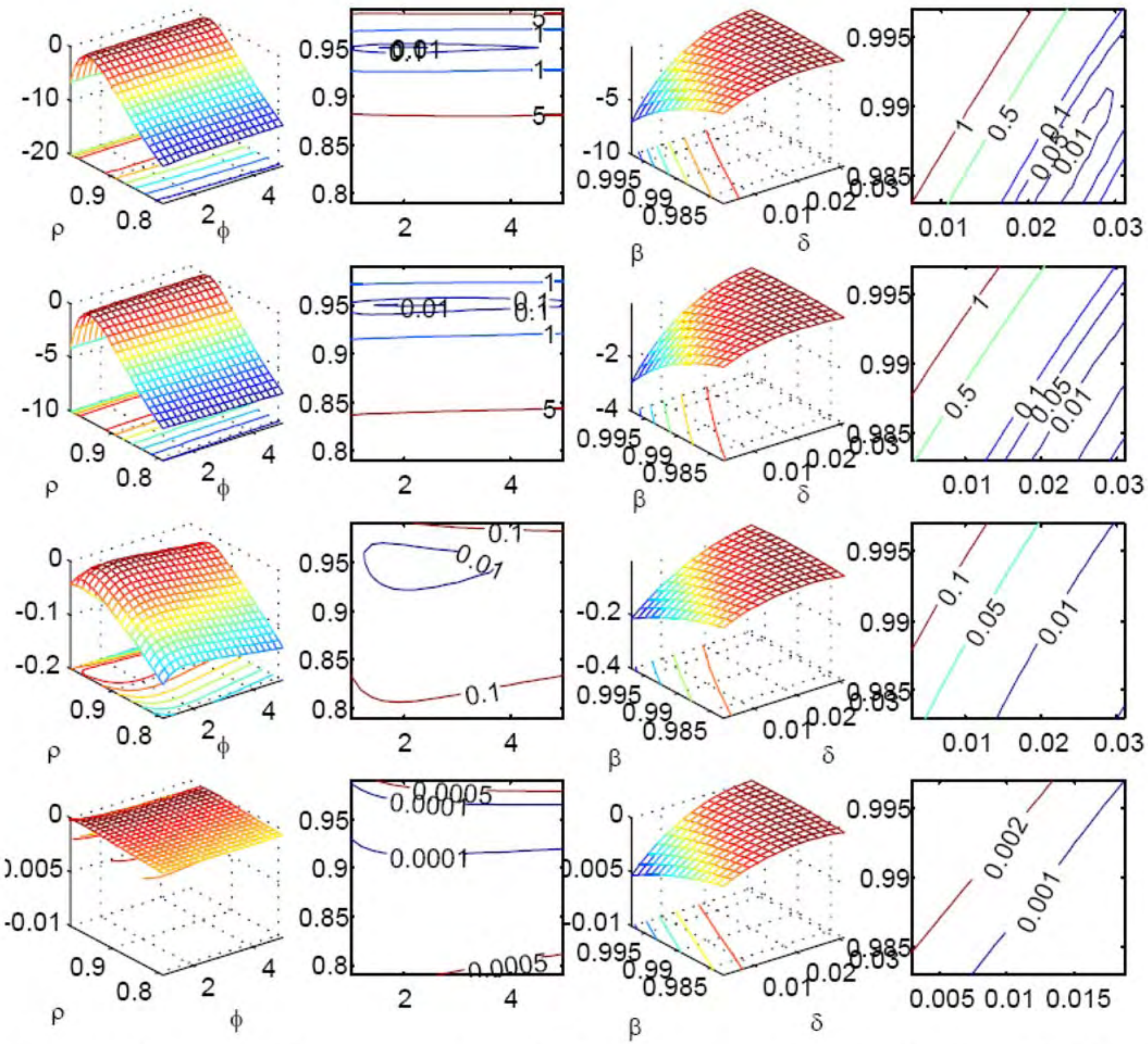

Figure 1: Distance surfaces and contours plot: Basic, Subset, Matching VAR and Weighted

5.1 Likelihood based estimation

Suppose y_t and c_t observed with measurement error

$$
\begin{aligned}
y_t^o &= y_t + \omega_1 v_{1t} && (29)\\
c_t^o &= c_t + \omega_2 v_{2t} && (30)\\
\mathbf{v}_t &= \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} \backsim NID(\mathbf{0}, \mathbf{I}_2) && (31)\\
Cov(\mathbf{v}_t, e_s) &= \mathbf{0} && (32)
\end{aligned}
$$

Hence we can write state space form:

$$\mathbf{x}_t = \begin{bmatrix} k_{t+1} \\ a_t \\ c_t \\ y_t \end{bmatrix}, \mathbf{y}_t = \begin{bmatrix} c_t^o \\ y_t^o \end{bmatrix} \quad (33)$$

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}e_t \quad (34)$$

$$\mathbf{y}_t = \mathbf{C}\mathbf{x}_t + \mathbf{D}\mathbf{v}_t \quad (35)$$

where

$$\mathbf{A} = \begin{bmatrix} \eta_{kk} & \eta_{ka} & 0 & 0 \\ 0 & \phi & 0 & 0 \\ \eta_{ck} & \eta_{ca}\phi & 0 & 0 \\ 1-\alpha & \alpha\phi & 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \eta_{ka}\sigma \\ \sigma \\ \eta_{ca}\sigma \\ \alpha\sigma \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{bmatrix} \quad (36)$$

Therefore matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ are all functions of structural parameters $\boldsymbol{\theta}$:

$$\boldsymbol{\theta} = \begin{bmatrix} g \\ \alpha \\ \delta \\ \beta \\ \gamma \\ \phi \\ \sigma \\ \omega_1 \\ \omega_2 \end{bmatrix} \tag{37}$$

The likelihood of the model can be obtained via application of the Kalman filter

$$p(\mathbf{y}_t|\underline{\mathbf{y}}_{t-1}, \boldsymbol{\theta}), t = 1, 2, ..., T \tag{38}$$

We can use a MH algorithm to simulate posterior distribution of the parameters or (ML approach)

5.2 Some toy results

- T=150
- Simulated data(?!!!)
- Bayesian priors
- main ingredients: `rbc_main.m`, `rbc_model_solution.m`, `dsge_kf.m`

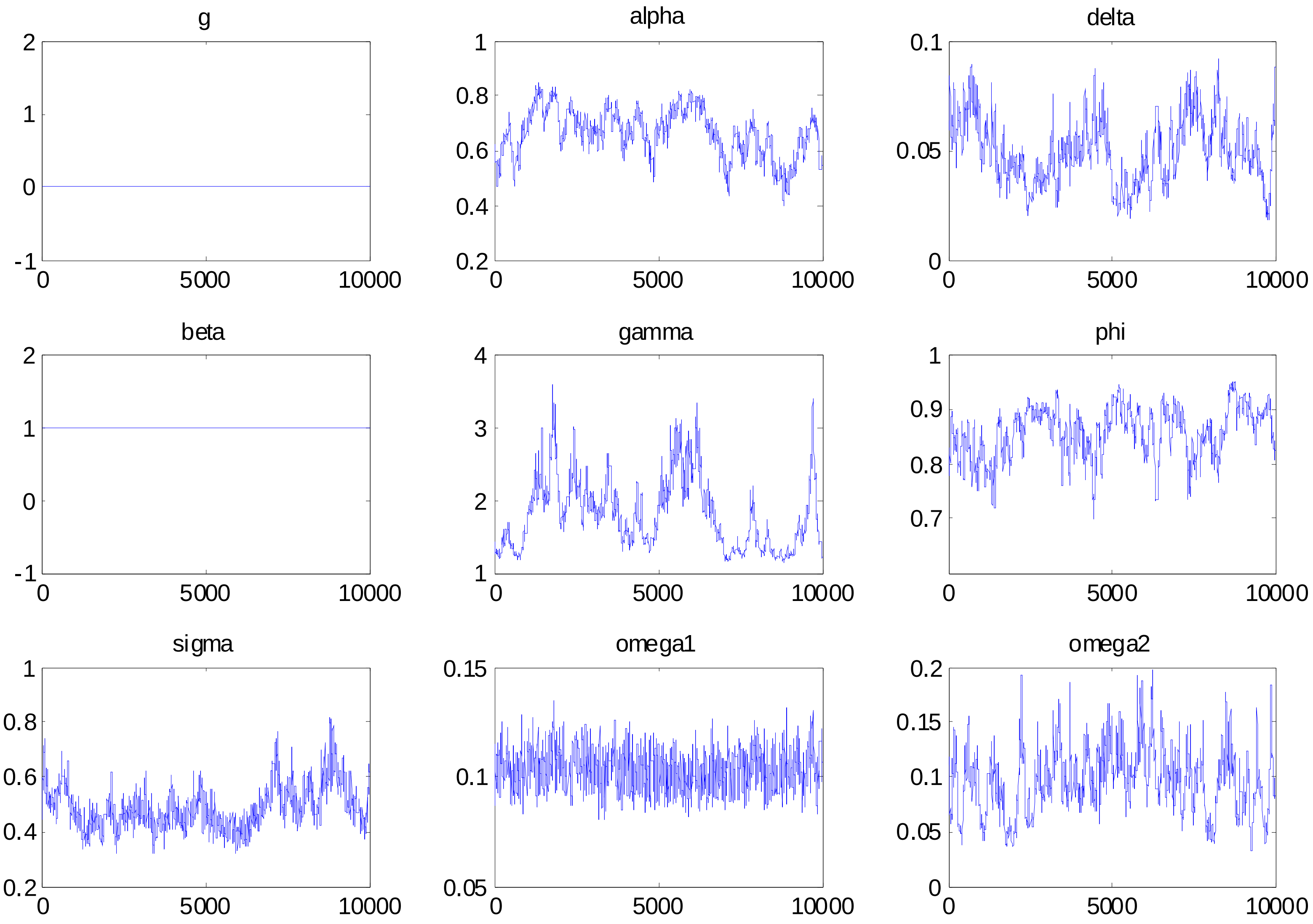
g
alpha
delta
beta
gamma
phi
sigma
omega1
omega2
0
5000
10000

Figure 3

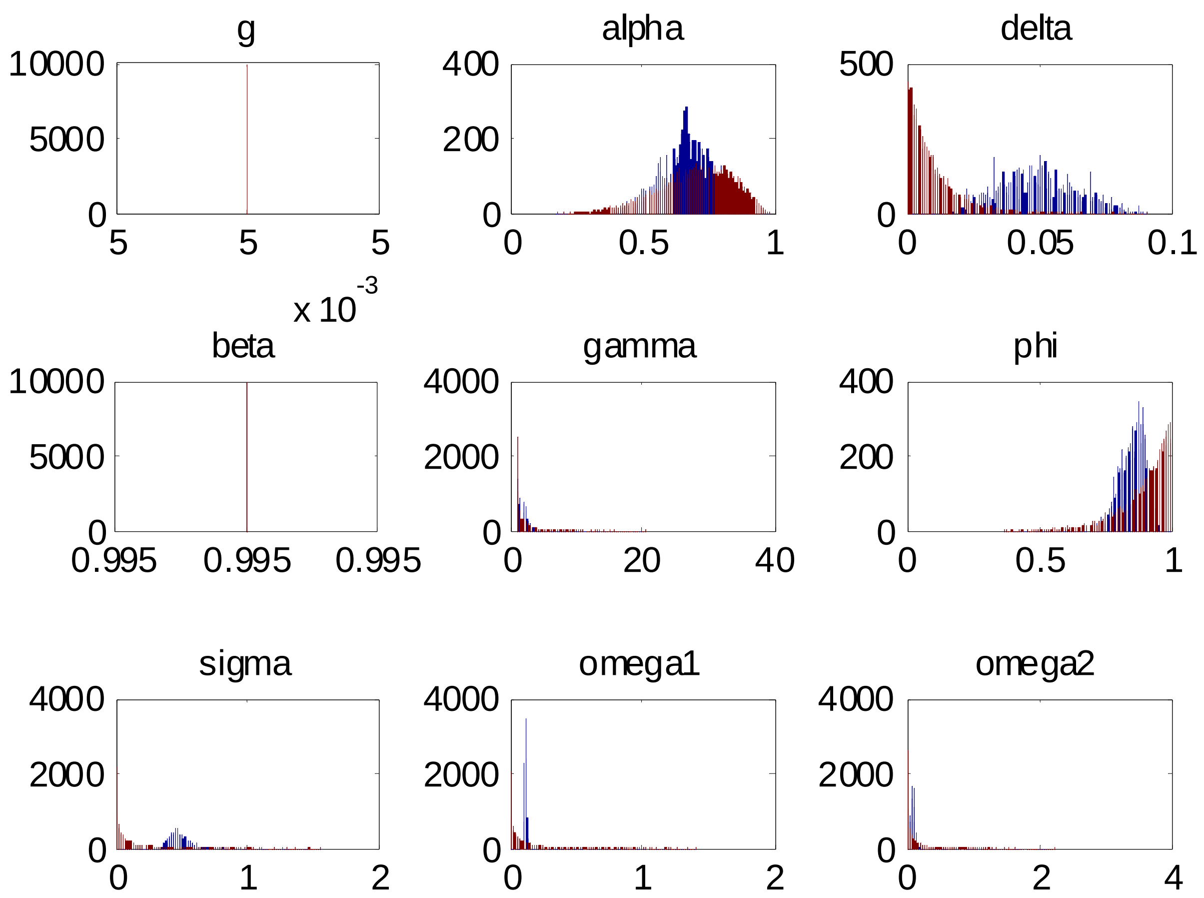
g
10000
5000
0
5
5
5
x 10^-3
alpha
400
200
0
0
0.5
1
delta
500
0
0
0.05
0.1
beta
10000
5000
0
0.995
0.995
0.995
gamma
4000
2000
0
0
20
40
phi
400
200
0
0
0.5
1
sigma
4000
2000
0
0
1
2
omega1
4000
2000
0
0
1
2
omega2
4000
2000
0
0
2
4

Figure 4

Table 1: DSGE results

	post m.	post sd	post low	post up	prior m	prior sd	prior low	prior up	cal
g	0.0050	0.0000	0.0050	0.0050	0.0050	0.0000	0.0050	0.0050	0.0050
α	0.6638	0.0858	0.4873	0.8087	0.7009	0.1377	0.4000	0.9249	0.6667
δ	0.0497	0.0146	0.0239	0.0792	0.0099	0.0098	0.0003	0.0364	0.0250
β	0.9950	0.0000	0.9950	0.9950	0.9950	0.0000	0.9950	0.9950	0.9950
γ	1.8143	0.4938	1.1859	2.9602	2.0070	1.4339	1.0009	6.1746	2.0000
ϕ	0.8579	0.0455	0.7621	0.9326	0.9001	0.0910	0.6665	0.9973	0.9000
σ	0.4878	0.0781	0.3668	0.6827	0.1001	0.1414	0.0001	0.4989	0.5000
ω_1	0.1028	0.0080	0.0883	0.1198	0.0998	0.1423	0.0001	0.5101	0.1000
ω_2	0.0962	0.0294	0.0430	0.1569	0.1012	0.1454	0.0001	0.4954	0.1000