Term structure in DSGE models: some theoretical and empirical issues

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DSGE models and term structure: a difficult marriage (I)

- Backus et al (1989): models with rep agent and time separable utility cannot generate sign, size and variability of risk premia
- Rudebusch and Swanson (2008): what happens when nominal frictions are included? Very little. Needed other features such recursive preferences (EZ) or habit (Wachter, 2006, Piazzesi and Schneider, 2006, in endowment economies)
DSGE models and term structure: a difficult marriage (II)

- Dual approach or combined model?
- Estimation of a satisfactory specification is not so easy.
Model features (I): preferences

- external habit in preferences \( h_t = C_{t-1} \)

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t - bh_t)^{1-\gamma}}{1 - \gamma} - \chi_0 \frac{l_t^{1+\chi}}{1 + \chi} \right),
\]

- Microfounded Nominal SDF

\[
m_{t,t+j} \equiv \beta^j \frac{(c_{t+j} - bC_{t+j-1})^{-\gamma}}{(c_t - bC_{t-1})^{-\gamma}} \frac{P_t}{P_{t+j}}.
\]
Model features (IIa): firms

- MC in firms producing intermediate goods ($f \in [0, 1]$)

$$y_t(f) = A_t k^{(1-\alpha)} l_t(f)^\alpha,$$

- aggregate tech shock

$$\log A_t = \rho A \log A_{t-1} + \varepsilon_t^A,$$
Model features (IIb): firms

- PC in final good sector with CES tech

\[ Y_t = \left[ \int_0^1 y_t(f)^{1/(1+\theta)} \, df \right]^{1+\theta}. \]

- Calvo contract expiring with probability \( 1 - \zeta \)
Model features (III): FOCs

- Intratemporal

\[ \frac{w_t}{p_t} = \frac{\gamma_0 l_t^\gamma}{(c_t - bC_{t-1})^{-\gamma}} \]

- Intertemporal

\[ (c_t - bC_{t-1})^{-\gamma} = \beta e^{it} E_t (c_{t+1} - bC_t)^{-\gamma} P_t / P_{t+1}, \]
Model features: public sector

- govt

\[ Y_t = C_t + \delta K + G_t, \]
\[ \log G_t = \rho_G \log G_{t-1} + \epsilon_t^G, \]
Model features: public sector

- Taylor rule

\[
i_t = \rho_i i_{t-1} + (1 - \rho_i)[1/\beta + \bar{\pi}_t + g_y(Y_t - \bar{Y})/\bar{Y} + g_\pi(\bar{\pi}_t - \pi^*)] + \epsilon_t^i,
\]

\[
\bar{\pi}_t = \theta_\pi \bar{\pi}_{t-1} + (1 - \theta_\pi)\pi_t,
\]
The SDF in action (I)

- All assets must satisfy SDF relationship

\[
p_t^{(n)} = E_t[m_{t+1}p_{t+1}^{(n-1)}]
\]

\[
i_t^{(n)} = -\frac{1}{n} \log p_t^{(n)}
\]
The SDF in action (II)

- consol with geometrically declining (rate $\delta_c$) coupon and Macauley duration $= n = 40$

\[
\tilde{p}_t^{(n)} = 1 + \delta_c E_t m_{t+1} \tilde{p}_{t+1}^{(n)},
\]

\[
\tilde{i}_t^{(n)} \equiv \log\left(\frac{\delta_c \tilde{p}_t^{(n)}}{\tilde{p}_t^{(n)} - 1}\right).
\]
The SDF in action (III)

- riskiness: price covaries with mg utility of c
- eg $\pi \uparrow \Rightarrow p \downarrow$; if $\pi$ and $y$ negatively correlated (eg supply shock), then very risky! very big RP.
Term premium (I)

- define first risk neutral price of the bond

\[
\hat{p}_t^{(n)} = E_t \sum_{j=0}^{\infty} e^{-i_{t,t+j}} \delta^j_c,
\]

\[
i_{t,t+j} = \sum_{n=0}^{j} i_{t+n}
\]

\[
\hat{p}_t^{(n)} = 1 + \delta_c e^{-i_t} E_t \hat{p}_{t+1}^{(n)}
\]
Term premium (II)

\[ \psi_t^{(n)} \equiv \log \left( \frac{\delta_c \tilde{p}_t^{(n)}}{\tilde{p}_t^{(n)} - 1} \right) - \log \left( \frac{\delta_c \tilde{p}_t^{(n)}}{\tilde{p}_t^{(n)} - 1} \right), \]

- RP
Other measures of long term bond risk (I)

- slope of TS
- excess one-period holding return

\[ x_t^{(n)} \equiv \frac{p_t^{(n-1)}}{p_t^{(n)}} - e^{i_{t-1}}. \]  
\[ \tilde{x}_t^{(n)} \equiv \frac{\delta_c \tilde{p}_t^{(n)} + e^{i_{t-1}}}{\tilde{p}_t^{(n)}} - e^{i_{t-1}}. \]  
\[ (ZC) \]  
\[ (consol) \]
Other measures of long term bond risk (II)

- measures based on Campbell and Shiller (1991) regression

\[ i_{t+1}^{(n-1)} - i_t^{(n)} = \alpha_{CS}^{(n)} + \beta_{CS}^{(n)} \frac{i_t^{(n)} - i_t}{n-1} + \epsilon_t^{(n)} \]
How to solve the model

- linear approximation would deliver
  \[ \hat{\rho}_t = \hat{\rho}_t \]
  (certainty equivalence of linear models)
- quadratic approximation would imply
  \[ \psi_t^{(n)} = \psi^{(n)} \]
- third order approx delivers time varying RP
Is this approach compatible with estimation?

- solution of the model: reduced form
- reduced form leads to state space and likelihood
- third order solution: 10 minutes
- estimation not feasible: only calibration
Table 1
Baseline parameter values and sensitivity analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline case value</th>
<th>Low case</th>
<th>High case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>Mean[$\psi^{(10)}$]</td>
<td>Value</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.7</td>
<td>0.013</td>
<td>0.85</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>0.014</td>
<td>0.995</td>
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<tr>
<td>$\theta$</td>
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<td>0.008</td>
<td>0.4</td>
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<tr>
<td>$\xi$</td>
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<td>0.026</td>
<td>0.9</td>
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<tr>
<td>$\gamma$</td>
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<td>-0.015</td>
<td>6</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>1.5</td>
<td>0.006</td>
<td>5</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.66</td>
<td>0.010</td>
<td>0.9</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.9</td>
<td>0.004</td>
<td>0.95</td>
</tr>
<tr>
<td>$\rho_C$</td>
<td>0.9</td>
<td>0.014</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma_A^2$</td>
<td>0.005²</td>
<td>0.006</td>
<td>0.02²</td>
</tr>
<tr>
<td>$\sigma_C^2$</td>
<td>0.002²</td>
<td>0.014</td>
<td>0.008²</td>
</tr>
<tr>
<td>$\rho_l$</td>
<td>0.73</td>
<td>0.038</td>
<td>0.9</td>
</tr>
<tr>
<td>$\delta_\pi$</td>
<td>0.53</td>
<td>-0.035</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_y$</td>
<td>0.93</td>
<td>0.035</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma_f^2$</td>
<td>0.004²</td>
<td>0.012</td>
<td>0.008²</td>
</tr>
<tr>
<td>$K/(4\bar{Y})$</td>
<td>2.5</td>
<td>1.25</td>
<td>5</td>
</tr>
<tr>
<td>$G/\bar{Y}$</td>
<td>0.17</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Memo:</td>
<td>$\chi_0$ 4.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\delta_c$ 0.9848</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Calibration II

<table>
<thead>
<tr>
<th>Variable</th>
<th>U.S. data 1960–2007</th>
<th>Parameterizations of DSGE model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Baseline</td>
</tr>
<tr>
<td>$sd[C]$</td>
<td>1.19</td>
<td>1.36</td>
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<tr>
<td>$sd[Y]$</td>
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<tr>
<td>$sd[L]$</td>
<td>1.71</td>
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<td>$sd[w^r]$</td>
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<td>2.27</td>
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<tr>
<td>$sd[\pi]$</td>
<td>2.52</td>
<td>2.35</td>
</tr>
<tr>
<td>$sd[i]$</td>
<td>2.71</td>
<td>2.06</td>
</tr>
<tr>
<td>$sd[r]$</td>
<td>2.30</td>
<td>1.97</td>
</tr>
<tr>
<td>$sd[i^{(10)}]$</td>
<td>2.41</td>
<td>0.55</td>
</tr>
<tr>
<td>Mean[$\psi^{(10)}$]</td>
<td>1.06</td>
<td>0.014</td>
</tr>
<tr>
<td>$sd[\psi^{(10)}]$</td>
<td>0.54</td>
<td>0.001</td>
</tr>
<tr>
<td>Mean[$i^{(10)} - i$]</td>
<td>1.43</td>
<td>-0.053</td>
</tr>
<tr>
<td>$sd[i^{(10)} - i]$</td>
<td>1.33</td>
<td>1.55</td>
</tr>
<tr>
<td>Mean[$x^{(10)}$]</td>
<td>1.76</td>
<td>0.014</td>
</tr>
<tr>
<td>$sd[x^{(10)}]$</td>
<td>23.43</td>
<td>6.98</td>
</tr>
<tr>
<td>$\rho^{(10)}_CS$</td>
<td>-3.49</td>
<td>0.97</td>
</tr>
</tbody>
</table>
Calibration III

- average term premium = 1.4 bps, std = .1 bps
- same with bigger models (CEE 2005, SW 2003)
- third order approx per se: not very useful
Pump up std errors

- some success but sacrifice fit of the macrovariables
- "estimate" the parameters
Habit formation a’ la CC (1999) I

Campbell and Cochrane (1999): long memory specification

\[ s_t \equiv \frac{c_t - bh_t}{c_t} \]

\[ h_t \equiv \frac{C_t(1 - S_t)}{b} \]

\[ \log S_t = (1 - \phi) \log \bar{S} + \phi \log S_{t-1} + \left( \sqrt{1 - 2 \log(S_t/\bar{S})} - 1 \right) [\log(C_t/C_{t-1}) - E_{t-1} \log(C_t/C_{t-1})] \]
Habit formation a’ la CC (1999) II

- still not satisfactory: RP below 1 bp
- Wachter (2006) instead has good results
- but Wachter has endowment economy
- in production economy consumers can use labor-consumption trade off to react to shocks
- Hence needed frictions in the labor market(?)
Quadratic adjustment costs I

- each household must pay

$$\kappa \left[ \log \left( \frac{l_t}{l_{t-1}} \right) \right]$$

- $\kappa$ price of insurance against shocks
Quadratic adjustment costs II

![Graph showing quadratic adjustment costs](image-url)
Quadratic adjustment costs III

- but needed very high adjustment costs: not big adj in c or labor supply
- will induce enormous variations in wages, mg costs, prices
- hence excessive volatility in wages and inflation
• Blanchard and Gali (2005): wage bargaining friction

\[
\log w_t^r = \mu \log w_{t-1}^r + (1 - \mu)(\log w_t^{r*} + \omega),
\]

• not big effects on RP (3.4 bps), while macro variables volatility is contained
Staggered nominal wage contracts

- Erceg et al (2000): households supplies labor with Calvo mechanism (CEE, SW)
- decreases RP!
- this is due to completeness of markets
No way out?

- these features do not work in a satisfactory way
- does it depend on structure or parameter values?
- are there other solutions/mechanisms?