Introduction to Bayesian econometrics for macroeconomists, Problem Set I

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The required data set are in:

`phdbocconi_ps1.zip`

available on the course web page

Exercise 1 Given the ULRM with Gaussian IID errors and exogenous regressors, suppose we have the following priors

$$\boldsymbol{\beta} \backsim \mathbf{N}(\underline{\boldsymbol{\mu}}_{\beta}, \underline{\mathbf{H}}_{\beta}^{-1}) \quad (0.1)$$

$$\underline{s} \cdot h \backsim \chi^2_{\underline{\nu}} \quad (0.2)$$

show that marginal posterior distribution of $\boldsymbol{\beta}$ does not have an analytically known form.

On the basis of the following alternative prior

$$\boldsymbol{\beta} \backsim \mathbf{N}(\underline{\boldsymbol{\mu}}_{\beta}, h^{-1}\underline{\mathbf{H}}_{\beta}^{-1}) \quad (0.3)$$

$$\underline{s} \cdot h \backsim \chi^2_{\underline{\nu}} \quad (0.4)$$

show that the marginal posterior distribution of $\boldsymbol{\beta}$ is Student-t.

Exercise 2 Suppose that in the ULRM with same properties as above, the researcher wants to impose prior information of the kind:

$$\mathbf{R}\boldsymbol{\beta} = \mathbf{d} + \boldsymbol{\varepsilon}_0 \quad (0.5)$$

$$\boldsymbol{\varepsilon}_0 \backsim N(\mathbf{0}, \underline{\mathbf{H}}_R^{-1}) \quad (0.6)$$

what is the posterior distribution of $\boldsymbol{\beta}$ conditional on h? What would happen to this distribution if the prior variance goes to zero?

Exercise 3 The file `IMP.XLS` contains seasonally adjusted quarterly data for Italy, 1970:1-1996:4. The variables contained in the file are:

- LM = log real import
- LY = log real GDP
- $LPPR$ = log PPI
- LPM = import deflator.

The equation we have in mind (demand for imports) is:

$$LM_t = \beta_1 + \beta_2 LY_t + \beta_3 LPPR_t + \beta_4 LPM_t + \beta_5 LM_{t-1} + \varepsilon_t \tag{0.7}$$

1. Supposing that error terms are normal IID, propose a prior for the parameters of the model and construct a Gibbs sampling algorithm to draw from posterion distribution. Give an estimate of posterior mean and variance - covariance matrix of these parameters.
2. Using confidence intervals based on the posterior distribution, verify the constraint $\beta_3 = -\beta_4$.
3. Using confidence intervals based on the posterior distribution, verify that the long run elasticity of imports with respect to output is unity ($\beta_2/(1-\beta_4) = 1$).
4. What is the posterior probability that β_5 is less than one? And the posterior probability that β_2 is positive?

Exercise 4 In the file `USinflation.xls` we have montly data for US inflation. Assume we have Gaussian IID noise. Specify a prior for all the parameters.

1. Estimate a AR(4) model with intercept for that series using a Gibbs Sampling approach. Report posterior mean, variance and 95% confidence intervals for each parameter and compare them with corresponding features of the prior distribution.
2. Obtain 1%, 5%, 10% 50%, 90%, 95%, 99% quantiles of the one step ahead predictive distribution of inflation.