Introduction to Bayesian econometrics for macroeconomists, Problem Set II

Gianni Amisano

16/02/06
The required data sets are in:
`phdbocconi_ps2.zip`
available on the course web page

Exercise 1 Suppose we can use a tri-variate VAR(3) model for the following series

$$
\begin{aligned}
r_t &= \text{real ex-post short term rate} \\
lu_t &= \ln\left(\frac{u_t}{1-u_t}\right), u_t = \text{unemployment rate} \\
\pi_t &= \text{inflation rate}
\end{aligned}
$$

Data are obtained via FRED II and contained in the data set `US_trivariate.xls` .

Use the first 20 years of observations (up to end of 1967) to estimate via OLS a constant parameter VAR model to calibrate the prior of a TVP VAR which will be estimated using the subsequent observations. Defend your choices in this regard.

Obtain a posterior simulation of the TVP VAR model and a posterior distribution for the NAIRU at the end of 1979 and the end of 2005. For the same dates provide the posterior mean and median of the degree of monetary activism, assuming the real rate reacts only to past values of lu and π. (For NAIRU use conditional projections 5 years ahead).

What is the posterior probability that the parameters at the end of the sample are such that the resulting VAR is stationary?

Note: you can use the code contained in the file `tvp_bvar.zip`. The program estimates a Bayesian TVP-VAR based on the following representation

$$\begin{aligned} \mathbf{y}_t &= (\mathbf{I}_n \otimes \mathbf{x}_t^{'})\boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t \qquad (0.1) \\ \boldsymbol{\beta}_t &= \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t \\ \begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\eta}_t \end{bmatrix} &\backsim NID\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{H}_{\varepsilon\varepsilon}^{-1} & 0 \\ 0 & \mathbf{H}_{\eta\eta}^{-1} \end{bmatrix} \right) \qquad (0.2) \end{aligned}$$

Exercise 2 Show which is the Kalman filtering recursion in the following separate cases:

1. $\begin{matrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\eta}_t \end{matrix}$ are instantaneusly correlated;

2. alternativelu, the matrix $\mathbf{H}_{\eta\eta}$ is parameterised as

$$\mathbf{H}_{\eta\eta} = \underset{(n\times n)}{\mathbf{H}_{\varepsilon\varepsilon}} \otimes \underset{(k\times k)}{\mathbf{H}} \qquad (0.3)$$

   where k is the number of parameters in each equation of the VAR. Discuss the implications of the results in this last case.

Exercise 3 In the file `sp500c.xls` we have daily returns on the SP500 Composite index and we want to estimate a 2 state Markow Switching model of the kind

$$y_t = c_{s_t} + \phi_{s_t} y_{t-1} + \sigma_{s_t} e_t \qquad (0.4)$$

$$e_t \backsim NID(0,1) \qquad (0.5)$$

and the latent state variable s_t has constant transition probabilities. Suppose you identify state 1 for being the low volatility state. Compute posterior probabilities of the parameters and the posterior probabilities that y_t was in the low volatility state at the end of March 2000, 2001, 2002. Compute posterior odds ratio of this model with respect to a model with no Markov Switching using all sample data. Is correlation of the levels significant? Is it different across states?

Show how you simulate the posterior density of the parameters and latent variables given the following model for the same series:

$$(y_t - \mu_{s_t}) = \phi_{s_t}(y_{t-1} - \mu_{s_{t-1}}) + \sigma_{s_t} e_t \qquad (0.6)$$

Exercise 4 Show what happens to the filtering recursion in a 2 state MS model when

$$p_{11} = 1 - p_{22}$$

and discuss the implications.

Exercise 5 Use the same DSGE model and the same series seen in class to

estimate the parameters (keep β and g at the calibrated value) and construct a posterior odds ratio for the hypothesis that the CRRA parameter is greater than 2.