Introduction to Bayesian econometrics for macroeconomists, Take home exam

Gianni Amisano (amisano@eco.unibs.it)

24/02/06. Due by 26/06/06 12.00 pm.
Data set and code are in:
`phdbocconi_takehome.zip`
available on the course web page.
The other Matlab programs distributed during the course and posted on the website are necessary.

You must submit your paper electronically by Sunday 26/02/06, midnight. Submit your paper AND the code you have used. Make sure you describe completely the procedures that you have followed and the assumptions you have used. If, for time constraints or any other reasons, you cannot carry out some computations, provide at least a concise explanation of how you would carry out the computations.

You can write your paper in English or Italian. You can use RTF, Microsoft Office, Sciword (ie .tex and .dvi) and pdf files. You can zip your files. Remember to write your complete name on your paper.

Exercise 1 Given the tri-variate VAR(3) model for the following series

$$
\begin{aligned}
r_t &= \text{real ex-post short term rate} \\
lu_t &= \ln\left(\frac{u_t}{1-u_t}\right), u_t = \text{unemployment rate} \\
\pi_t &= \text{inflation rate}
\end{aligned}
$$

contained in the data set `US_trivariate.xls` (same data set as in PS2).

The main code to be used is `main_tvp_bvar2.m` which is included. The program estimates a Bayesian TVP-VAR based on the following representation

$$\mathbf{y}_t = (\mathbf{I}_n \otimes \mathbf{x}_t^{'})\boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t, \tag{0.1}$$

$$\mathbf{x}_t = \left[\mathbf{y}_{t-1}^{'}, \mathbf{y}_{t-2}^{'}, \mathbf{y}_{t-3}^{'}\right]^{'} \tag{0.2}$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t$$

$$\begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\eta}_t \end{bmatrix} \backsim NID\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{H}_{\varepsilon\varepsilon}^{-1} & 0 \\ 0 & \mathbf{H}_{\eta\eta}^{-1} \end{bmatrix} \right) \tag{0.3}$$

It uses the first 20 years of observations (up to end of 1967) to estimate via OLS a constant parameter VAR model. This is done to calibrate the prior on $\mathbf{H}_{\varepsilon\varepsilon}$ and to initialise the Kalman filter. The prior on $\mathbf{H}_{\eta\eta}$ is Wishart with parameters $\left(k^* + 2, \frac{10^{-7}}{k^*+2} \times \mathbf{I}_{k^*}\right)$ where k^* is the dimension of $\boldsymbol{\beta}_t$.

1. Discuss the setting of the priors and the initialisation of the Kalman Filter.
2. Using data from 1968 to 2005 obtain a posterior simulation of the model and comment on the properties of the realised simulation output.
3. Obtain the posterior distribution for the NAIRU and the core inflation at the end of 1980 and the end of 2005. For NAIRU and core inflation use respectively $E_t u_{t+h}$ and $E_t \pi_{t+h}$ where h means 5 years ahead. (hint: use a simulation based approach).
4. Describe what happens to the Kalman filter and the Carter and Cohn recursions when $\mathbf{H}_{\eta\eta}$ is a matrix of zeros.

Exercise 2

1. Using data on inflation and unemployment from the previous exercise, estimate a linear regression model of the kind

$$\pi_t = c + \phi_1 \pi_{t-1} + \phi_2 \pi_{t-2} + \gamma_1 u_{t-1} + \gamma_2 u_{t-2} + h^{-1/2} e_t \tag{0.4}$$

$$e_t \backsim NID(0, 1) \tag{0.5}$$

by using a reasonable prior of your choice on the parameters (discuss and motivate your choice).

2. Provide a Bayesian test of the hypothesis that the Phillips curve relationship implied by the equation above is vertical in the long run.

3. Obtain posterior distribution of the NAIRU implied by the above relationship and test hypothesis that NAIRU is below 5%.

4. Suggest (and implement if possible) an importance sampling procedure to estimate model (0.4).

Exercise 3

1. Estimate a version of equation (0.4) in which all parameters vary according to an unobserved Markov discrete 2-state latent variable. Identify state 1 as the state with the smallest persistence probability.

2. Obtain a series of the posterior probabilities that the latent state was equal to 1 at each observation in the sample. How do these probability relate to the filtered probabilities?

3. Suggest how the MCMC approach should be modified if the transition probabilities depended on a set of exogenous regressors $\mathbf{z}_t$ according to the following specification

$$p_{ii,t} = p(s_t = i|s_{t-1} = i) = \Phi(\boldsymbol{\gamma}_i^{'}\mathbf{x}_t), i = 1, 2 \quad (0.6)$$

$$\Phi(a) = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-\omega^2}{2}\right) d\omega \quad (0.7)$$

(this is a bit hard but at least try to figure it out!).

Exercise 4

1. Use the same DSGE model and the same series seen in class to estimate the

   parameters (keep β and g at the calibrated value). Discuss the quality of the simulator output.

2. In your view, how should the Kalman filtering procedure be initialised?

3. Compute a Bayesian test of the hypothesis that the instantaneous utility function is logarithmic.

4. Compute a Bayesian posterior odds ratio for the hypothesis that the AR parameter is greater than .95.